Theoretical Aspects of Gravitational Lensing in T*e*V*e* S

Mu-Chen Chiu

Department of Physics, National Central University, Chung-Li, Taiwan 320, R.O.C.

chiumuchen@gmail.com

Chung-Ming Ko

Institute of Astronomy, Department of Physics and Center for Complex System, National Central University, Chung-Li, Taiwan 320, R.O.C.

cmko@astro.ncu.edu.tw

Yong Tian

Department of Physics, National Taiwan University, Taipei, Taiwan 106, R.O.C.

yonngtian@gmail.com

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ABSTRACT

Since Bekenstein's (2004) creation of his Tensor-Vector-Scalar theory (T*e*V*e* S), the Modified Newtonian dynamics (MOND) paradigm has been redeemed from the embarrassment of lacking a relativistic version. One primary success of T*e*V*e* S is that it provides an enhancement of gravitational lensing, which could not be achieved by other MONDian theories. Following Bekenstein's work, we investigate the phenomena of gravitational lensing including deflection angles, lens equations and time delay. We find that the deflection angle would maintain its value while the distance of closest approach vary in the MOND regime. We also use the deflection angle law to derive magnification and investigate microlensing light curves. We find that the difference in the magnification of the two images in the point mass model is not a constant such as in GR. Besides, microlensing light curves could deviate significantly from GR in the deep MOND regime. Furthermore, the scalar field, which is introduced to enhance the deflection angle in T*e*V*e* S, contributes a negative effect on the potential time delay. Unfortunately this phenomenon is unmeasurable in lensing systems where we can only observe the time delay between two images for a given source. However, this measurable time delay offers another constraint on the mass ratio of the dark matter and MOND scenarios, which in general differs from that given by the deflection angle. In other words, for a lensing system, if two masses, m_{gN} and m_{gM}, are mutually alternatives for the deflection angles in their own paradigm, regarding the time delay they are in general in an exclusive relation.

Subject headings: gravitational Lensing — MOND — dark matter — gravitation — relativity

1. Introduction

While Newtonian gravitational theory was applied to the extragalactic region, it no longer passed the trials as easily as it had done in solar system. Within Coma and Virgo cluster the velocities of individual galaxies are so large that its total mass must exceed the sum of masses of its shining galaxies if the clusters are gravitational bound systems (Zwicky 1933; Smith 1936). The 21 cm emission line from HI of spiral galaxies also show the need of extra masses besides visible stars

(van Albada et al. 1985; Begman 1989). Moreover, X-ray observations and strong gravitational lensing in early type galaxies and clusters imply that there are 60 $\sim$ 85% masses in some undetected form (Fabbiano et al. 1989; Böhringer 1995; Loewenstein & White 1999). This so-called "missing mass" problem may be explained in two different ways: the existence of Dark Matter, and a modification of Newton law , hence of General Relativity (GR). The most famous representative of the modified gravity theory party is Modified Newtonian Dynamics (MOND) proposed by Milgrom in 1983 (Milgrom 1983a,b,c), which assumes that Newton's second law needs to be modified when the acceleration is very small:

$$\tilde{\mu}(|\mathbf{a}|/\mathfrak{a}_0)\mathbf{a} = -\boldsymbol{\nabla}\Phi_{\rm N}. \qquad (1)$$

Here $\Phi_{\rm N}$ is the usual Newtonian potential of the visible matter and $\mathfrak{a}_0 \approx 1 \times 10^{-10}$ m s^{-2} from the empirical data, such as the Tully-Fisher relation and rotation curves (Sanders & Verheijen 1998); $\tilde{\mu}(x) \approx x$ for $x \ll 1$, $\tilde{\mu}(x) \to 1$ for $x \gg 1$. In the solar system where accelerations are strong compared to $\mathfrak{a}_0$, the formula (1) is just Newton's second law $\mathbf{a} = -\boldsymbol{\nabla}\Phi_{\rm N}$.

From this simple formulation the Tully-Fisher relation and flat rotation curves follow trivially (Sanders & McGaugh 2002). The rotation curves of spiral galaxies, especially of LSBs, can be explained extremely well (Begeman et al. 1991; Sanders 1996; McGaugh & de Blok 1998; Sanders & Verheijen 1998). Moreover, the dearth of dark matter in some ordinate elliptical galaxies can be intrinsically explained in MOND paradigm (Milgrom & Sanders 2003). Even though many such phenomena in galaxy systems can be explained by this simple modification with an astonishing precision, the MOND paradigm had lacked a flawless relativistic gravitational theory for two decades (Bekenstein & Milgrom 1984; Bekenstein 1988; Sanders 1988, 1997), and thus could not address the lensing and other relativistic gravitational observations. However, since the end of March 2004, the embarrassment has been removed by Bekenstein (2004), when he finally contrived a Lorentz covariant relativistic gravitational theory called T*e*V*e*S, which is based on three dynamical gravitational fields: an Einstein metric $g_{\mu\nu}$, a vector field $\mathfrak{U}_\mu$ and a scalar field ϕ (the acronym T*e*V*e*S is for this Tensor-Vector-Scalar content). In this scheme, T*e*V*e*S contains MOND for nonrelativistic dynamics, agrees with the solar system measurements for the β and γ PPN coefficients and avoids superluminal propagation of the metric, scalar and vector waves. Moreover, while concerning cosmogony, it not only has cosmological models which are similar to that of GR but also overcomes the Silk damping problem of the CDM-dearth universe with the help of the scalar field (Skordis et al. 2005). Now, we can investigate the relativistic MOND paradigm on many observations and experiments (Giannios 2005; Skordis et al. 2005; Zhao 2005a,b; Zhao et al. 2005).

On the other hand, the GR with Dark Matter (DM) paradigm, despite its success for the large scale structure and the cosmic microwave background (CMB)(see e.g. Spergel et al. 2003), has some problems on the galactic scales. For example, the dark matter halos, if they really exist, must have a shallower core than the universal NFW density profile had predicted (cuspy core problem, see e.g. Navarro & Steinmetz 2000; Romanowsky et al. 2003); the disc size will be too small while considering the angular transformation from baryon to dark matter halo (angular momentum catastrophe, see e.g. Navarro et al. 1995; Moore et al. 1999; Maller & Dekel 2002; Kazantzidis et al. 2004); the number of sub-halos predicted by non-baryon simulation is much more than the observed number of dwarf galaxies (substructure problems, see e.g. Kauffmann et al. 1993; Navarro et al. 1995). Despite these problems, the GR with DM paradigm is less falsifiable and supported by a larger group because of its unlimited choice of models and parameters.

In order to judge which side better accounts for the phenomena of the natural world, the disparity between T*e*V*e*S and GR with DM paradigms has to be closely investigated. This distinction will be important for the physics of MOND and the "missing mass" problem. Gravitational lensing (GL) phenomena provide us an opportunity because they dominate on scales that even surpass the size of dark matter halo (Kochanek 2004).

For a long time, light bending has been a great challenge for Modified Gravity. Any scalar field jointly coupling with the Einstein metric via a conformal transformation would yield the same optical phenomena as in the standard Einstein metric. Thus in the context of MOND, where little dark

matter exists, it is hard to produce the anomalously strong deflection of photons (Bekenstein & Sanders 1994; Soussa & Woodard 2003, 2004). However, this problem can be solved by postulating the disformal relation between the physical metric and the Einstein metric (Sanders 1997). This is what Sanders' stratified theory, and then T*e*V*e*S take over.

In this paper, we will discuss the theoretical aspects of Gravitational lensing systems within the framework of T*e*V*e*S. In § 2, we summarize the characteristics of T*e*V*e*S and introduce a cosmology model within which gravitational lensing phenomena will be discussed. In § 3 we derive the deflection angle in the general form and its two particular limits: the Newtonian and the MOND regimes. From this we obtain lens equations, which are the foundation of the other GL phenomena. In § 4, we apply the standard time delay formalism in T*e*V*e*S. Finally, § 5 summarizes the presuppositions of this paper and discusses some possible conclusions of our results in § 3 and § 4.

2. Fundamentals of T*e*V*e*S and The Cosmological Model

Before introducing the action of T*e*V*e*S, we should explain some important features of the disformal metric relation. In Bekenstein's Tensor-Vector-Scalar theory (T*e*V*e*S), the role of Einstein's metric $g_{\alpha\beta}$ is replaced by the physical metric (Bekenstein 2004)

$$\tilde{g}_{\alpha\beta} = e^{-2\phi}(g_{\alpha\beta} + \mathfrak{U}_\alpha \mathfrak{U}_\beta) - e^{2\phi}\mathfrak{U}_\alpha \mathfrak{U}_\beta, \qquad (2)$$

with a normalization condition on the vector field:

$$g^{\alpha\beta}\mathfrak{U}_\alpha \mathfrak{U}_\beta = -1. \qquad (3)$$

The physical metric, which governs the dynamical behavior of matter (the equivalence principle i.e. minimal coupling) is simultaneously influenced by a 4-vector field $\mathfrak{U}_\alpha$, a scalar field ϕ and the Einstein's gravity $g_{\alpha\beta}$.

From the theoretical viewpoint, if DM does not exist in galaxies or cluster scale, the concept of geodesics must be different from that of GR. Although there is no deeper theoretical base for the disformal relation (2), empirical testament like lensing effects will be important to justify this choice (Sanders 1997).

The cosmological model, which determines the background spacetime structure, is an essential ingredient in gravitational lensing (Schneider et al. 1992; Petters, Levine & Wambsganss 2001). On the one hand, the distance in the gravitational lensing scenario would vary under different cosmological models. On the other hand, gravitational lensing provides a tool to determinate H_0 and to constrain the cosmological parameters (Blandford & Narayan 1986; Schneider 1996). Since we have used an alternative gravitation theory, it is important to discuss its effects on cosmology, which is solely influenced by gravity. In this section, we introduce Bekenstein's Relativistic MOND theory, and then describe the revised Friedman model.

2.1. Actions of T*e*V*e*S

The convention in this paper are the metric signature +2 and units with $c = 1$.

There are three dynamical fields $g_{\alpha\beta}$, $\mathfrak{U}_\alpha$ and ϕ built in T*e*V*e*S. In addition to these fields, there is another non-dynamical scalar field σ, which is introduced into the scalar field action to eliminate the superluminal problem of ϕ (Bekenstein 2004).

By varying the actions for these fields, the respective equations of motion for the scalar fields, vector fields and the Einstein metric can be arrived at, therefore the physical metric is determined.

The action in T*e*V*e*S can be divided into four parts, $S = S_g + S_s + S_v + S_m$. The geometrical action,

$$S_g = (16\pi G)^{-1} \int g^{\alpha\beta} R_{\alpha\beta}(-g)^{1/2} d^4x, \qquad (4)$$

is identical to the Einstein-Hilbert action in GR and is used to produce the Einstein metric, $g_{\alpha\beta}$. The matter action coupling to the scalar field, ϕ, and vector field, $\mathfrak{U}_\alpha$, through $\tilde{g}_{\alpha\beta}$ is taken to be

$$S_m = \int \mathcal{L}(-\tilde{g})^{1/2} d^4x, \qquad (5)$$

where $\mathcal{L}$ is a lagrangian density for the fields under consideration and should be considered as functionals of the physical metric and its derivatives.

The vector action with the form

$$\begin{aligned} S_v \quad = \quad & -\frac{K}{32\pi G}\int \big[g^{\alpha\beta}g^{\mu\nu}\mathfrak{U}_{[\alpha,\mu]}\mathfrak{U}_{[\beta,\nu]} \\ & -2(\lambda/K)(g^{\mu\nu}\mathfrak{U}_\mu \mathfrak{U}_\nu + 1)\big](-g)^{1/2}d^4x \end{aligned} \qquad (6)$$

yields the vector field equation. Here, K is a dimensionless parameter, and λ is a Lagrange multiplier. Moreover, $\mathfrak{U}_{[\alpha,\mu]}$ means $\mathfrak{U}_{\alpha,\mu} - \mathfrak{U}_{\mu,\alpha}$.

The action of the two scalar fields is taken to have the form

$$\begin{aligned} S_s &= -\tfrac{1}{2}\int \big[\sigma^2 h^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} \\ &\quad +\tfrac{1}{2}G\ell^{-2}\sigma^4 F(kG\sigma^2)\big](-g)^{1/2}d^4x, \quad (7) \end{aligned}$$

where $h^{\alpha\beta} \equiv g^{\alpha\beta} - \mathfrak{U}^\alpha\mathfrak{U}^\beta$; F is a free function in order to produce the dynamical behaviors of MOND, which needs to be constrained empirically from cosmological model; l is a constant length and k is another dimensionless parameter in this theory. In the FRW-like cosmological models, k is smaller than 10^{-2} (Bekenstein 2004; Skordis et al. 2005). Variation of σ of the action gives

$$-\mu F(\mu) - \tfrac{1}{2}\mu^2 F'(\mu) = y \quad (8)$$

where $y = k\ell^2 h^{\mu\nu}\phi_{,\mu}\phi_{,\nu}$ is a free function of μ equivalent to F. However, in later discussions, it is more convenient to consider $\mu(y)$ as a function of y. Variation of ϕ of the action then gives

$$\begin{aligned} &\left[\mu\,(y)\,h^{\alpha\beta}\phi_{,\alpha}\right]_{;\beta} = \\ &\qquad kG\big[g^{\alpha\beta} + (1+e^{-4\phi})\mathfrak{U}^\alpha\mathfrak{U}^\beta\big]\tilde{T}_{\alpha\beta}. \quad (9) \end{aligned}$$

As σ through $\mu = kG\sigma^2$ can be expressed in terms of ϕ explicitly, Eq. (9) involves ϕ only.

To illustrate ideas, in this work we adopt the free function suggested by Bekenstein (2004)

$$y = \frac{3}{4}\frac{\mu^2(\mu-2)^2}{1-\mu}, \quad (10)$$

whence y approaches zero asymptotically as $y \sim 3\mu^2$ when $\mu \to 0$, and $y \to \infty$ when $\mu \to 1$.

We are going to discuss gravitational lensing of a point mass in § 3 and § 4. Thus we are interested in static and spherically symmetric geometry. In this case μ runs from 1 (the Newtonian regime) to 0 (the deep MOND regime). In order to satisfy the behavior of the potential at these limits we require $y \propto \mu^2$ asymptotically as $\mu \to 0$, and $y \to \infty$ as $\mu \to 1$. In the intermediate regime $y(\mu)$ (or $\mu(y)$) is arbitrary. Nonetheless, the exact functional form does not affect our results because we do not consider the intermediate regime.

2.2. The Cosmological Model

As pointed out by Bekenstein (2004), the physical metric of FRW cosmology in T*eV*e S is obtained by replacing $dt \to e^{\phi}dt$ and $a \to e^{-\phi}a$ in the primitive Roberson-Walker metric. Here, the Friedmann equation becomes:

$$\frac{\dot{\tilde{a}}}{\tilde{a}} = e^{-\phi}\left(\frac{\dot{a}}{a} - \dot{\phi}\right), \quad (11)$$

where $\dot{\tilde{a}}/\tilde{a}$ and $\dot{a}/a$ are essentially equal in the whole history because $e^{-\phi} \sim 1$ and $\dot{\phi} \sim 0$ (Bekenstein 2004; Hao & Akhoury 2005).

Since we have the additional scalar fields and vector fields in Bekenstein's picture, the Einstein equation, which tells us the geometrical properties, is not only determined by the energy-momentum tensor as before, but also by the scalar and vector fields. Here, ϕ, σ, and $\mathfrak{U}_\alpha$ are assumed to partake of the symmetry of the spacetime. The vector field is considered to be parallel with the cosmological time, i.e. $\mathfrak{U}^\alpha = \delta^\alpha_t$, because there should be no preferred spatial direction (Bekenstein 2004). Moreover, if we apply Weyl's postulate, which states that galaxies would move along nonintersecting world lines and therefore the spatial components in the energy-momentum tensor can be neglected (Weyl 1923), then only the tt component of Einstein's equation remains:

$$\begin{aligned} G_{tt} &= 8\pi G(\tilde{\rho}e^{-2\phi} + \sigma^2\dot{\phi}^2) \\ &\quad + \frac{2\pi}{k^2\ell^2}\mu^2 F(\mu) + \Lambda \\ &= 8\pi G\tilde{\rho}e^{-2\phi} + \frac{4\pi}{k^2\ell^2} \\ &\qquad \left[\frac{3}{2}\mu^2 F(\mu) + \frac{1}{2}\mu^3 F'(\mu)\right] + \Lambda. \quad (12) \end{aligned}$$

The appearance of Λ in Eq. (12) results from the choice of the free function $F(\mu)$, which is decided by Eq. (8) and Eq. (10). It is also possible to construct another form of $F(\mu)$ that makes the dark energy as an effective phenomenon of the scalar field ϕ (Hao & Akhoury 2005). However, before having any theoretical reason for the choice of the free function $F(\mu)$, we would like to leave the interpretation alone and only treat it as a parameter in the cosmological model.

By the Roberson-Walker metric, which determines the tt component of the Einstein tensor

Eq. (12), we can get the modified Friedmann equation in a flat universe in the form

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_M + \frac{8\pi G}{3}\rho_\phi + \frac{\Lambda}{3}, \tag{13}$$

where $\rho_M = \tilde{\rho}e^{-2\phi}$ corresponds to the matter density in Einstein's picture and

$$\rho_\phi = \frac{3}{2Gk^2\ell^2}\left[\tfrac{3}{2}\mu^2 F(\mu) + \tfrac{1}{2}\mu^3 F'(\mu)\right] \tag{14}$$

is the density due to the scalar field. For convenience, we can rewrite Eq. (13) in terms of Ω density:

$$H^2 = H_0^2[\Omega_m(z) + \Omega_\phi(z) + \Omega_\Lambda(z)]. \tag{15}$$

Since gravitational lensing must be observed within the framework of galaxies or clusters of galaxies, we only need to consider the cosmological model in the matter and Λ-dominated era, so $\Omega_m(z) \simeq \Omega_m(1+z)^3$. Furthermore, in order to avoid a significant effect of the integrated Sachs Wolfe term on CMB power spectrum, $\Omega_\phi(z)$ must be extremely small compared with the matter density (Skordis et al. 2005). So even if we do not know exactly how $\Omega_\phi(z)$ evolves with z, we can neglect it, because its value is overwhelmed by the uncertainty of Ω_m and Ω_Λ for a non-CDM universe (McGaugh 1999, 2004; Skordis et al. 2005).

To summarize, in TeVeS the angular distance at any redshift can be calculated by

$$d_A \simeq \frac{(1+z)^{-1}}{H_0}\int_0^z dz\left[\Omega_m(1+z)^3 + \Omega_\Lambda\right]^{-1/2}. \tag{16}$$

We conclude that if there is any difference in the angular distance between GR and TeVeS, it arises from the low matter density universe in TeVeS rather than the influence of the scalar field ϕ.

Finally, it is worth noting that although it is not trivial to consider an inhomogeneous universe on the gravitational lens scale (Dyer & Roder 1973), we do not consider that case because we only deal with a single point mass lens embedded in a background Friedmann universe in this paper.

3. Gravitational Lensing in TeVeS

When building gravitational lensing models, people usually use some approximations, such as the static and thin lens approximations. We also adopt the weak field assumption since all known gravitational lens phenomena are influenced by weak gravitational systems. Although the Schwarzschild lens (point mass model) is not sufficient for gravitational lensing effects, we consider this for its simplicity. Moreover it gives results of the same order-of-magnitude as those for more realistic lenses (Schneider et al. 1992).

3.1. Deflection Angle in Terms of Mass

In the weak field limit, the physical metric of a static spherically symmetric system in TeVeS can be expressed in the isotropic form:

$$\begin{aligned}\tilde{g}_{\alpha\beta}\, dx^\alpha dx^\beta &= -(1+2\Phi)dt^2 \\ &+(1-2\Phi)[d\varrho^2 + \varrho^2(d\theta^2 + \sin^2\theta d\varphi^2)],\end{aligned} \tag{17}$$

where $\Phi = \Xi\Phi_N + \phi$ and $\Xi \equiv e^{-2\phi_c}(1+K/2)^{-1}$ with ϕ_c as the asymptotic boundary value of ϕ and $K < 10^{-3}$ from the constrain of PPN parameters (Bekenstein 2004).

Considering a quasistatic system with a perfect fluid, the scalar equation (9) can be reduced into

$$\nabla\cdot\Big[\mu(y)\nabla\phi\Big] = kG\tilde{\rho}, \tag{18}$$

because $\mathfrak{U}_\alpha$ has only a time-independent temporal component and $\phi \ll 1$ in this case. Here $y = k\ell^2(\nabla\phi)^2$, which is obtained from the general form, $y = k\ell^2 h^{\mu\nu}\phi_{,\mu}\phi_{,\nu}$, in spherical symmetric geometry. From Eq. (18), we can find a relation between Φ_N and ϕ,

$$\nabla\phi = (k/4\pi\mu)\,\nabla\Phi_N. \tag{19}$$

Therefore, Φ can be determined whenever μ is known.

It is also worth to stress that $\tilde{g}_{tt} = -(1+2\Phi)$ follows the empirical definition of the gravitational mass m_g in the solar system when $\mu \to 1$:

$$g = \frac{1}{2}\frac{\partial\tilde{g}_{tt}}{\partial\varrho} = (\Xi + k/4\pi)\frac{Gm_g}{\varrho^2} = \frac{G_N m_g}{\varrho^2}, \tag{20}$$

and that the spatial components, $\tilde{g}_{ii} = (1-2\Phi)$, guarantee the theory passes the classical post-Newtonian (PPN) tests of gravity (Bekenstein 2004).

We now apply the static spherically symmetric metric, c.f. Eq. (17), to the GL systems. Considering a light ray, which propagates on a null

geodesic, and moves in the equatorial plane, i.e. $\theta = \pi/2$, we can arrive at three constants of motion from the physical metric in equation (17) with some proper affine parameter, τ:

$$(1+2\Phi)\dot{t} = E, \quad (21)$$
$$(1-2\Phi)\varrho^2\dot{\varphi} = L, \quad (22)$$
$$(1-2\Phi)\dot{\varrho}^2 + (1+2\Phi)\varrho^{-2}L^2 - (1-2\Phi)E^2 = 0, \quad (23)$$

where over dot denotes the derivative with respect to τ. Eliminating E and L in Eq. (23) from Eq. (21) and (22), and recalling the fact that $\dot{\varrho}$ vanishes at the closest approach, $\varrho = \varrho_0$, yields

$$b^2 \equiv \frac{L^2}{E^2} = \frac{\varrho_0^2(1-2\Phi_0)}{(1+2\Phi_0)}, \quad (24)$$

where, $\Phi_0 \equiv \Phi(\varrho_0)$. Combining Eq. (24) and the three constants of motion, gives the equation which describes the shape of the photon orbit:

$$-(1-4\Phi)+(1-4\Phi_0)(\varrho_0/\varrho)^2[\varrho^{-2}(d\varrho/d\varphi)^2+1] = 0. \quad (25)$$

The solution of this equation in quadrature is

$$\varphi = \int^{\varrho} \left\{ (\frac{\varrho}{\varrho_0})^2 \left[1 - 4(\Phi - \Phi_0)\right] - 1 \right\}^{-1/2} \frac{d\varrho}{\varrho}. \quad (26)$$

If we take the Taylor expansion of Eq. (26) to the first order of Φ, then it is easy to see that the quadrature is a combination of the angle of a straight line feeling no gravity and the angle of the deviation due to the gravitation field. Hence the deflection angle can be approximated from the first order terms of this Taylor expansion:

$$\Delta\varphi = \varrho_0 \int_{\varrho_0}^{\varrho} \frac{2\varrho(\Phi-\Phi_0)}{(\varrho^2-\varrho_0^2)^{3/2}} d\varrho. \quad (27)$$

Furthermore, after some manipulations (see e.g. Bekenstein 2004), the deflection angle is

$$\Delta\varphi = \varrho_0 \int_{\varrho_0}^{\varrho} \frac{4\Phi'}{(\varrho^2-\varrho_0^2)^{1/2}}\, d\varrho - \varrho_0 \frac{4\Phi}{(\varrho^2-\varrho_0^2)^{1/2}}. \quad (28)$$

It is obvious that the deflection angle between the closest approach and any position is always positive. First, gravity is always attractive (therefore $\Phi' > 0$), so the value of the quadrature is positive because every term in it is positive. Second, the second term of the right hand side is negligible even in the MOND regime, in which Φ behaves as $\ln\varrho$, if ϱ is large.

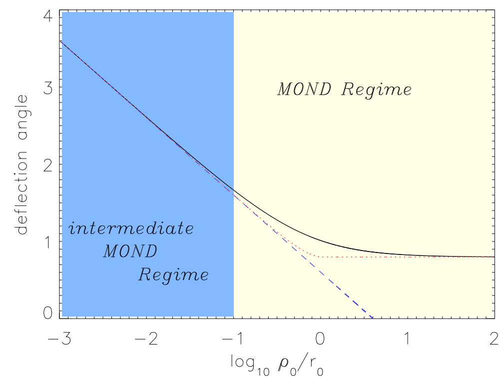

Fig. 1.— Deflection angle of light derived under the point mass model in Te*Ve*S with $k = 0.01$ and $K = 0.01$(solid line) and GR (dashed line). The closest approach is normalized to the length scale r_0, which depends on the mass of the lens. The dotted line shows the work of Mortlock and Turner (2001), who started from a reliable intuition without a relativistic MOND. In the two extreme limits, their result is exactly the same as the deflection law derived in Te*Ve*S. However, in the intermediate region there is a difference.

3.2. Two Limits of the Deflection Angle

Even though we can make sure that the deflection angle can be enhanced in Te*Ve*S by the simple argument above, we need the exact form of the the potential to reach the real value of a deflection angle. Unfortunately, the form of potential depends on the free function $F(\mu)$, which can only be constrained empirically. However, there are two limits, the Newtonian and the MOND regime, in which the dynamical behavior has been clearly known, and so to the potential form. Actually, any choice of the $F(\mu)$ cannot contradict the empirical potential form in these two limits. Here, we discuss the deflection angles in the two regimes (see Fig. 1).

The MOND regime means $\nabla\phi$ is significant compared with $\nabla\Phi$, whence by Eq. (19), $\mu \ll 1$ has to be a sufficient condition for this regime. From Eq. (10) and $y = k\ell^2(\nabla\phi)^2$, we can obtain $\mu \approx (k/3)^{1/2} l|\nabla\phi|$ when $\mu \ll 1$ (i.e., in MOND regime). It has to be mentioned that although this relation results from the freedom of an arbitrary function, $\mu \sim |\nabla\phi|$ should always persist in

order to guarantee $\nabla\phi$ varying as $1/\varrho$ in the deep MOND regime, a condition which follows from the MOND paradigm. Using this relation in Eq. (19) to eliminate $\nabla\phi$ and replacing $\nabla\Phi_N$ with $\nabla\Phi$ gives

$$\mu = (k/8\pi\Xi)\left(-1+\sqrt{1+4|\Phi'|/\mathfrak{a}_0}\,\right) = \frac{k}{4\pi}\left(\frac{|\Phi'_N|}{\Xi\mathfrak{a}_0}\right)^{1/2}. \tag{29}$$

Here, the constant

$$\mathfrak{a}_0 \equiv \frac{(3k)^{1/2}}{4\pi\Xi\ell} \tag{30}$$

can be identified as Milgrom's constant. We should keep in mind that this form of μ is valid under the condition $\Phi' \ll (4\pi/k)^2\mathfrak{a}_0$. It also offers a criterion for distinguishing the Newtonian and the MOND regime.

We define a distance,

$$r_0 \equiv \left(\frac{G_N m_g}{\mathfrak{a}_0}\right)^{1/2}, \tag{31}$$

at which acceleration, $\mathbf{a}$, equals to Milgrom's constant, $\mathfrak{a}_0$. Then a particle is in the deep MOND regime if $\varrho \gg kr_0/4\pi$, and in the Newtonian regime if $\varrho \ll kr_0/4\pi$.

3.2.1. The Newtonian regime

Although a photon moving in a GL system may come from a distance $\varrho \gg kr_0/4\pi$ to the closest approach $\varrho_0 \ll kr_0/4\pi$ and then fly away again, the influence of gravity only dominates in the region which is very near the gravitation center. Therefore we call a GL system as being in the Newtonian regime if $\varrho_0 \ll kr_0/4\pi$.

In the Newtonian regime, Φ' in Eq (28) can be calculated from the relation $\Phi = \Xi\Phi_N + \phi$ and Eq. (19) for $\mu \to 1$. This would yield the deflection angle of a Schwarzschild lens,

$$\Delta\varphi = \frac{4Gm_g}{\varrho_0}(\Xi + \frac{k}{4\pi}) = \frac{4G_N m_g}{\varrho_0}. \tag{32}$$

Here again, G_N is identical to the gravitational constant measured in a local experiment. This is the same as the result from GR.

3.2.2. The Deep MOND Regime

In the deep MOND regime, $\varrho_0 \gg kr_0/4\pi$, μ has the form of Eq. (29). Then along with Eq. (19)

Fig. 2.— Embedding diagram of light bending. For a thin lens, the lensing equation and geometrical time delay can be immediately read off.

and Eq. (29), the deflection angle for a point mass model in the deep MOND regime can be arrived at from Eq. (28):

$$\Delta\varphi = \frac{4G_N m_g}{\varrho_0(\Xi + k/4\pi)} \cdot \left\{\Xi + \frac{\pi}{2}\varrho_0\left[\frac{\mathfrak{a}_0(\Xi^2 + k\Xi/4\pi)}{G_N m_g}\right]^{1/2}\right\}, \tag{33}$$

with $\Xi \equiv 1 - K/2 - 2\phi_c$. Since all of K, ϕ_c and k are much less than 1, for a given mass, the deflection angle in the deep MOND regime differs from that in GR (or equivalence in the Newtonian regime; Eq. [32]) by an amount almost independent of the distance of the closest approach.

We have to stress that even though we do not know exactly how deflection angle varies with respect to the closest approach in the intermediate MOND regime, our result based on the deep MOND assumption (i.e., $\mu \ll 1$), which is incorrect in the regime $\varrho_0 \le kr_0/4\pi$, approaches to the Newtonian prediction while ϱ_0 decreasing (see Fig. 1). Therefore we believe even in the intermediate regime, the deflection law based on the deep MOND assumption is approximately correct.

3.3. Magnification and Microlensing

From the observational point of view, the measurable data is not deflection angles but positions of the projected sources, i.e. $\theta \equiv \varrho_0/D_L$. Here, D_L is the angular distance of lens (Eq. [16]). Therefore we need a relation to connect θ and $\Delta\varphi$, the deflection angle derived in the last section. We can obtain their relation from an embedding dia-

gram (Fig. 2) and get the lens equations:

$$\beta = \theta_+ - \alpha(\theta_+), \tag{34}$$
$$\beta = \alpha(\theta_-) - \theta_- . \tag{35}$$

Here the reduced deflection angle is defined as $\alpha(\theta) = (D_{LS}/D_S)\Delta\varphi(\theta)$. In GR it is not important to distinguish the difference between Eq. (34) and Eq. (35) in finding θ Paczyński (1986). It is, however, not the case in T*eV*e S. In order to keep $\theta_+ = \theta_-$ when $\beta \to 0$, we must use Eq. (34) for θ_+ and Eq. (35) for θ_-.

After substituting $\Delta\varphi$ and θ into the lens equation Eq. (34), we get

$$\beta = \theta - \frac{D_{LS}}{D_L D_S}\frac{4G_N m_g}{\theta} \cdot [\frac{\Xi}{\Xi + k/4\pi} + \frac{\pi}{2}\frac{\theta D_L}{r_0}(\Xi + \frac{k}{4\pi})^{-1/2}]. \tag{36}$$

For clarity, we can define $\theta_0 \equiv r_0/D_L$ and θ_E with the form

$$\theta_E = \left(4G_N m_g \frac{D_{LS}}{D_L D_S}\right)^{1/2}. \tag{37}$$

θ_E is called the Einstein radius; it is used as a length scale in the standard GL modeling. As $k \ll 1$ and $K \ll 1$, the lens equation (36) can be reduced to

$$\beta \simeq \theta - \frac{{\theta_E}^2}{\theta}[1 + \frac{\pi}{2}\frac{\theta}{\theta_0}] \tag{38}$$

with the reduced deflection angle, $\alpha(\theta)$, as

$$\alpha \simeq \frac{{\theta_E}^2}{\theta}[1 + \frac{\pi}{2}\frac{\theta}{\theta_0}]. \tag{39}$$

If Eq. (35) is used instead of Eq. (35), then the only difference in Eq. (38) is from β to $-\beta$. It is easily seen that the effective contribution of the scalar field can be expressed as $\pi{\theta_E}^2/2\theta_0$ and is independent of θ_0 (or equivalently ϱ_0).

For a point source, there are always two projected images, which can be obtained from Eq. (34) and Eq. (35). The solutions are

$$\theta_\pm = \pm\frac{1}{2}\left\{\left(\beta \pm \frac{\pi\theta_E^2}{2\theta_0}\right) \pm \left[\left(\beta \pm \frac{\pi\theta_E^2}{2\theta_0}\right)^2 + 4\theta_E^2\right]^{1/2}\right\} > 0 \tag{40}$$

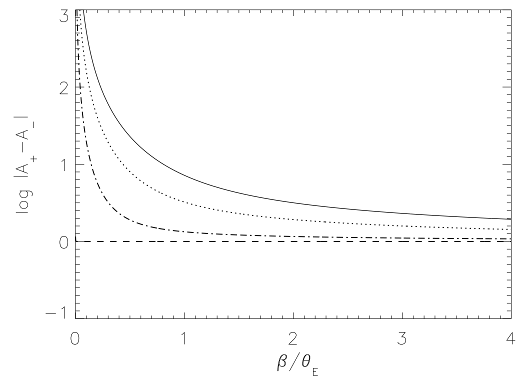

Fig. 3.— The difference in the magnification of the two projected images. In T*eV*e S, it deviates from the constant value in GR (dashed line). From the solid to dotted-dashed line, they represent the cases of T*eV*e S with $\theta_E/\theta_0 = 1.0$, 0.5 and 0.1.

Moreover, since gravitational lensing conserves surface brightness, the magnifications of the image to source intensity are determined by their area ratio,

$$A_\pm = \left|\frac{\theta_\pm}{\beta}\frac{\partial\theta_\pm}{\partial\beta}\right|. \tag{41}$$

Interesting enough, the difference in the magnification of the two images is no longer constant in T*eV*e S as in GR (see Fig. 3):

$$|A_+ - A_-| \geq 1 \tag{42}$$

And in the deep MOND regime, the total magnification is given by

$$A \equiv A_+ + A_- = \left(\frac{u_1}{2u}\right)\left[\frac{{u_1}^2 + 2}{u_1\,({u_1}^2+4)^{1/2}} + 1\right] \tag{43}$$
$$+ \left(\frac{u_2}{2u}\right)\left[\frac{{u_2}^2 + 2}{u_2\,({u_2}^2+4)^{1/2}} - 1\right], \tag{44}$$

where

$$u \equiv \frac{\beta}{\theta_E}\ ;\ u_1 = u + \frac{\pi}{2}\frac{\theta_E}{\theta_0}\ ;\ u_2 = u - \frac{\pi}{2}\frac{\theta_E}{\theta_0}. \tag{45}$$

The magnification reduces to the standard GR form as θ_0 (or r_0) $\to \infty$. It is then straightforward to study the light curves of microlensing (Paczyński 1986).

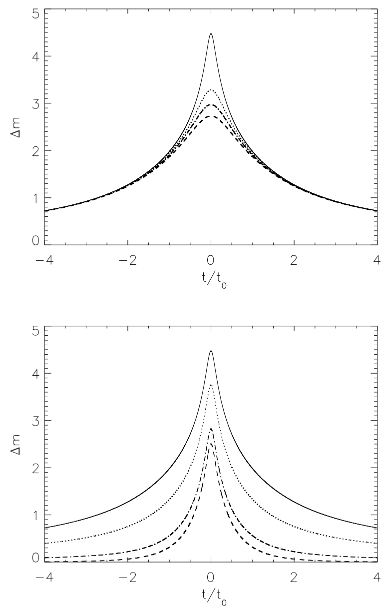

Fig. 4.— **upper panel:** Microlensing light curves in TeVeS for $\theta_E/\theta_0 = 1.0$. From solid line to dashed line, $\beta_{min}/\theta_E = 0.1$, 0.3, 0.4, 0.5. **lower panel:** Light curves of microlensing in TeVeS with $\theta_E/\theta_0 = 0.1$ (dashed-dotted), 0.5 (dotted), 1.0 (solid) and in GR (dashed line). The difference of the magnitude is given by $\Delta m = 2.5\ \log A$ and t_0 is the time that sources pass through Einstein radius.

Microlensing events results from the relative proper motions of lenses to the sources. Because of proper motions, the alignment between lens-observer and source-observer would vary with time,

$$u(t) = \sqrt{(\beta_{min}/\theta_E)^2 + (t/t_0)^2}, \qquad (46)$$

and so does the magnification. Here β_{min} occurs at the greatest of the alignment and t_0 is the time it takes the source to move with respect to the lens by one θ_E. Putting Eq. (46) into Eq. (44) and Eq. (45), we can obtain the microlensing light curves expressed in stellar magnitudes ($\Delta m \equiv 2.5 \log A$). Like the case in GR, the variation reaches the maximum as $\beta(t) = \beta_{min}$, and for any given $\pi{\theta_{_E}}^2/2\theta_{_0}$, the smaller the β_{min}, the greater amplitude of the light curve. However, the shape of light curves in TeVe S would deviate from that in GR as $\pi{\theta_{_E}}^2/2\theta_{_0}$ is significant (i.e., in deep MOND regime). In general the contribution of $\pi{\theta_{_E}}^2/2\theta_{_0}$ would raise the peak (see Fig. 4).

Even though the differences in microlensing events between TeVe S and GR only occurs at source redshift $z_s \geq 1$ (Mortlock & Turner 2001), which is beyond the current micolensing projects within the local group (see e.g., Alcock et al. 2000; Paczyński 1996), observations such as several quasar microlensing events (see e.g., Wambsganss 2001) may help us to decide whether MOND is a viable alternative to the dark matter paradigm.

4. Time Delay

Time-delay always plays an important role in gravitational lensing. It not only provides a more convenient way to get the lens equation (Schneider 1985; Blandford & Narayan 1986) but also has some important values in its own right (for more complete reviews, see Courbin et al. 2002 or Kochanek & Schechter 2003). As it is an alterative to the cold dark matter, it should be very interesting to study the behavior of time delay in TeVe S.

4.1. Arrival Time in TeVe S

We apply the static and isotropic metric described by Eq. (17) and assume that the photons are confined to the equatorial plane without loss of generality . Then, following the standard procedure of light bending (see e.g. Weinberg 1972), not only the deflection angle can be derived, but also the equation which describes the arrival time of a light ray

$$(1 - 4\Phi)\dot{\varrho}^2 = [1 - (\frac{1 + 4\Phi}{1 + 4\Phi_0})(\frac{\varrho_0}{\varrho})^2]\dot{t}^2. \qquad (47)$$

This equation has a quadrature as a solution, which states that a photon from ϱ_0 to any distance ϱ (or from ϱ to the closest approach distance ϱ_0) would require a time,

$$t = \int_{\varrho_0}^{\varrho} \frac{(1 - 4\Phi)^{1/2} d\varrho}{[1 - (1 + 4\Phi)/(1 + 4\Phi_0)(\frac{\varrho_0}{\varrho})^2]^{1/2}}. \qquad (48)$$

Taylor expanding Eq. (48) to the first order of $\mathcal{O}(\Phi)$, the original quadrature can be reduced to

$$t(\varrho, \varrho_0) = \int_{\varrho_0}^{\varrho} d\varrho [1 - (\frac{\varrho_0}{\varrho})^2]^{-1/2}$$

$$+\int_{\varrho_0}^{\varrho} d\varrho \frac{2(\Phi-\Phi_0)(\varrho_0/\varrho)^2}{[1-(\frac{\varrho_0}{\varrho})^2]^{3/2}}$$

$$-\int_{\varrho_0}^{\varrho} d\varrho \frac{2\Phi}{[1-(\frac{\varrho_0}{\varrho})^2]1/2} \quad (49)$$

The second quadrature of Eq. (49) differs from Eq. (27) only by a factor ϱ_0, whence, from this part, the arrival time contributions due to both gravitational mass and the scalar field are positive. However, this is not the case in the third quadrature of Eq. (49). Since Φ_N and ϕ share an opposite sign in Φ, their contributions to the resultant arrival time are also opposite. Moreover since the third quadrature is always larger than the second one (it is easily to observe this by multiplying by a factor "$1-(\varrho_0/\varrho)^2$" in both numerator and denominator of the second quadrature), the time delay due to the scalar field will always be opposite to that due to the gravitational mass, i.e. with an opposite sign. In other words, the arrival time will be shorten rather than enhanced by the positive scalar field; this is a necessary condition in T*eV*e S for not violating causality (Bekenstein 2004).

4.2. Arrival Time and the MOND Regime

As in the case of the deflection angle, in order to calculate time delay from Eq. (49), we need some additional information on the free function, $F(\mu)$ to determine the potential Φ in T*eV*e S. However, there are some differences in these two considerations. The first one arises from the fact that time delay is influenced by gravity even far away from the center. This makes a purely Newtonian regime unavailable. Therefore, we only consider the MOND regime here. Moreover, unlike the deflection angle case, we need information about Φ, in addition to Φ' for the time delay. Eliminating μ of Eq. (19) from Eq. (29) yields

$$\phi = (\Xi \mathfrak{a}_o G m_g)^{1/2} \ln \frac{4\pi\varrho}{k r_0} + \tilde{\phi}_c, \qquad (50)$$

where $\tilde{\phi}_c$ can be absorbed into the symmetric physical metric $\tilde{g}_{\mu\nu}$ by rescaling the t and ϱ coordinates appropriately (Bekenstein 2004). Recalling that $\Phi = \Xi\Phi_N + \phi$, and putting Eq. (49) into Eq. (50), we get the arrival time in the MOND regime:

$$t(\varrho,\varrho_0) = (\varrho^2-\varrho_0^2)^{1/2} + 2\Xi G m_g \left(\frac{\varrho-\varrho_0}{\varrho+\varrho_0}\right)^{1/2}$$

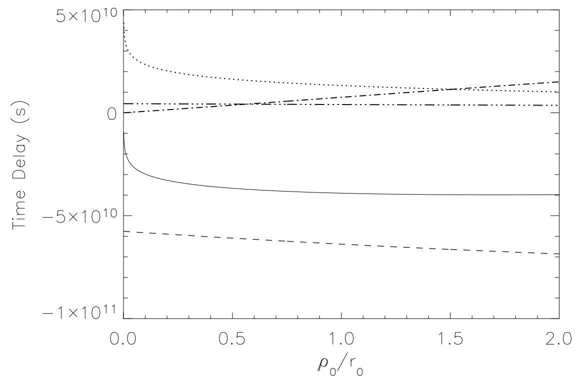

Fig. 5.— The time delay (the real light path minus the straight between sources and observers) of a lensing system with a $M = 5\times10^{14} M_\odot$ lens in the MOND regime. The closest approach is normalized to the length scale r_0 and both the observer and the source are located at 10 r_0. The dotted line and the dashed line show the potential time delay induced by the ordinary matter and scalar field respectively. Moreover the geometrical parts are represented by the dotted-dashed (scalar field) and dotted-dotted-dashed line(visible matter). Adding all components yields the total time delay (solid line) and shows a negative effect on the arrival time in T*eV*e S.

$$+2\Xi G m_g \ln(\frac{\sqrt{\varrho^2-\varrho_0^2}+\varrho}{\varrho_0})$$
$$-2\left(\Xi\mathfrak{a}_0 G m_g\right)^{1/2}$$
$$\left[(\ln(\frac{4\pi\varrho}{k r_0})-1)(\varrho^2-\varrho_0^2)^{1/2} + \varrho_0 \sec^{-1}(\frac{\varrho}{\varrho_0})\right]$$
$$+2\left(\Xi\mathfrak{a}_0 G m_g\right)^{1/2}$$
$$\left[\varrho_0 \sec^{-1}(\frac{\varrho}{\varrho_0}) - \varrho_0^2 \ln(\frac{\varrho}{\varrho_0})(\varrho^2-\varrho_0^2)^{-1/2}\right] .(51)$$

Here again, G differs by a factor of $\Xi + k/4\pi$ from the usual Newtonian constant, which is measured in the solar system experiments. We would like to stress that as a particular case of our cinsideration in § 4.1, the sum of last two terms in Eq. (51) is always negative.

Intrinsically, we may think that time delay (photon deviation of the path length) should increase with deflection angle as read off from the embedding diagram (Fig.2). However, this is only true if we do not consider the deflection poten-

tial. Actually, a positive scalar field will offer a hyperbolic-like spacetime, in which the distance is shorter than that of a flat universe (see Fig. 5).

4.3. Measurable Time Delay

Since it is the positions of sources and lens, not the distance from sources (or observers) to lens that are observed, time delay is conventionally expressed in terms of these dimensionless angles. It is appropriate to rewrite Eq. (49) as:

$$t = \int_{\varrho_0}^{\varrho} dl - \int_{\varrho_0}^{\varrho} 2\Phi dl, \tag{52}$$

here, $dl = [1 - (1 + 4\Phi/1 + 4\Phi_0)(\frac{\varrho_0}{\varrho})^2]^{-1/2} d\varrho$ is a line element like that of Euclidean geometry, i.e. it obeys $dl^2 = d\varrho^2 + \varrho^2 d\varphi^2$. Traditionally, we call the first quadrature in Eq. (52) the geometrical arrival time because it can be read off in a geometrical way (see Fig. 2):

$$t_{geom}(\varrho_1, \varrho_2) = \sqrt{(\varrho_0 - \eta)^2 + D_{LS}^2} + \sqrt{\varrho_0^2 + D_L^2}. \tag{53}$$

Here $\eta = D_S \beta$. Indeed if we let a photon move from ϱ_2 to ϱ_0 and from ϱ_0 to ϱ_1, it can be proved that the contributions to the arrival time calculated from the first two integrals of Eq. (49) are identical to the geometrical arrival time, c.f. Eq. (53). On the other hand, the third quadrature in Eq. (49), which equals the second integral of Eq. (52), contributes to what is called the *potential time delay*. Adding the geometrical and potential contributions to the arrival time and subtracting the arrival time for an unlensed ray from S to O, and considering the expansion of the cosmological background, we obtain the time delay of a possible ray compares to an un-deflected ray,

$$\triangle t = (1+z)\left[\frac{D_L D_S}{2D_{LS}}\left(\frac{\varrho_0}{D_L} - \frac{\eta}{D_S}\right)^2 - \psi(\varrho_0)\right] + \mathbb{B}. \tag{54}$$

where, $\psi(\varrho_0)$ called the *deflection potential* can be calculated from the potential time delay. Once again we only consider what happens in the MOND regime. For any given sources and lenses, the distances from the lenses to the observer (or sources) are decided, whence the potential time delay, which is given from the second and the fifth terms in Eq. (51), can be reduced to some constant plus the deflection potential contributions

$$\psi_{GR} = 4G_N m_g \ln \varrho_0, \tag{55}$$

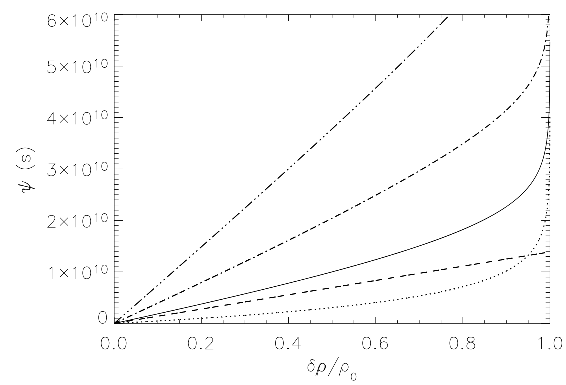

Fig. 6.— The measurable time delay (time delay between two images for a given source) of a gravitational lensing system in T*e*V*e* S. The system is assumed to have its largest closest approach at $\varrho_0 = 2\,r_0$ and a lens with $M = 5 \times 10^{14} M_\odot$ to resemble a cluster of galaxies alike Virgo cluster. The dotted line is the contribution from the visible mass, and the dashed line expresses the contribution from the scalar field. Their combination is showed by the solid line. When $\delta\varrho/\varrho_0 \to 0$, the discrepancy between the time delay of the scalar field and ordinary matter decreases. On the other hand, when $\delta\varrho/\varrho_0 \to 1$, the influence of the ordinary matter increases and ultimately exceeds that of the scalar field. For the same lens, two systems with $\varrho_0 = 5\,r_0$ and $\varrho_0 = 10\,r_0$ respectively are presented by dotted-dashed and dotted-dotted-dashed line.

$$\psi_\phi = 4\left(\frac{\mathfrak{a}_0 G_N m_g}{1 + k/4\Xi\pi}\right)^{1/2}\left(\frac{\pi}{2}\varrho_0\right), \tag{56}$$

$$\psi_{corr} = -\frac{4G_N m_g}{1 + 4\pi\Xi/k} \ln \varrho_0. \tag{57}$$

We have shown in § 4.2 that the time delay (relative to the undeflected path) is reduced rather enhanced in the T*e*V*e* S framework. Unfortunately, this phenomena cannot be measured in any GL system. For any given source at position β, we are only able to measure the time difference for two images $\varrho_0^{(1)}$, $\varrho_0^{(2)}$, which share the same lens potential, lens-observer, and lens-sources distances. In other words, the constant $\mathbb{B}$ in Eq. (54) is the same for all light rays from the source plane to the observer. Therefore in regard to the potential time delay, we can only measure the contributions

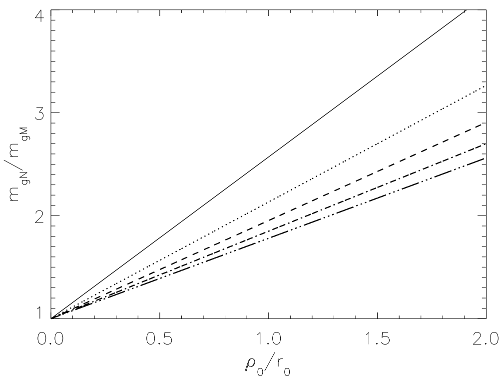

Fig. 7.— Mass ratios of the MOND and dark matter paradigms. The solid line shows the constraint on the mass ratio while requiring an identical amount of deflection angle in the two different paradigm. This constrain is found again from the requirement of the measurable time delay as $\delta\varrho \ll \varrho_0$. However, the corresponding mass ratios of the time delay will be increasingly less than that of the deflection angle as the difference between the two closest approach increases. From the dotted line to the dotted-dotted-dashed line, $\varrho_0^1/\varrho_0^2 =$ 2 ,3, 4, 5.

from the deflection potential, which possesses the same sign in GR and T*e*V*e* S (Fig. 6). Moreover, since the deflection potential does not depend on the length scale $kr_0/4\pi$, the free choice of $\hat{\phi}_c$ has no influence on the measurable time delay in GL systems.

Even though we cannot measure the opposite contribution to the arrival time in T*e*V*e* S, time delay between two images due to the deflection potential offers another constraint on the mass ratio, which is not a *priori* identical to that due to deflection angle.

In order to illustrate this, we consider a general case: the value of ϱ_0^1 is "n" times ϱ_0^2, whence the time delay between the two images is

$$\Delta\psi = 4G_N m_g \left[\ln n + \frac{\pi}{2}\frac{(n-1)}{n}\frac{\varrho_0^1}{r_0}\right]. \qquad (58)$$

For the extreme case, the difference between ϱ_0^1 and ϱ_0^2 is negligible compared to either distance. Hence, $\ln(\varrho_0^1/\varrho_0^2) \simeq \delta\varrho_0/\varrho_0$, and Eq. (58) can be reduced to:

$$\Delta\psi = 4G_N m_g \frac{\delta\varrho_0}{\varrho_0}\left(1 + \frac{\pi}{2}\frac{\varrho_0}{r_0}\right). \qquad (59)$$

If we compare Eq. (59) with Eq. (39), we can find that the similarity of the two equations makes the mass ratio m_{gN}/m_{gM} exactly the same under the requirement of either identical deflection angle or time delay. However, if the difference between ϱ_0^1 and $varrho_0^2$ is not small, the mass ratio given by the time delay will be less than that obtained from the deflection angle (Fig. 7).

4.4. Time Delay and Lensing Equation

As showed by Schnieder (1985) and Blandford and Narayan (1986), time delay with Fermat's principle offers an alternative for deriving the lens mapping. Moreover, since Fermat's principle has proved valid under a very general metric (Kovner 1990; Perlick 1990), we should be able to re-derive lens equation from our time delay result. Fermat's principle asserts that a light path is true if and only if its arrival time or, what amounts to the same, its time delay is stationary with respect to variation of the turning point. In other words, $\partial(\triangle t)/\partial\varrho_0 = 0$. With this sufficient and necessary condition for Fermat's principle, we can obtain the lens equation in the MOND regime,

$$\eta = \frac{D_S}{D_L}\varrho_0 - D_{LS}\nabla\psi. \qquad (60)$$

$\nabla\psi$ is composed of three parts; and when ϱ is very large

$$\nabla\psi_{GR} = 4\frac{G_N m_g}{\varrho_0} \qquad (61)$$

$$\nabla\psi_{\phi} = 2\pi\left(\frac{\mathfrak{a}_0 G_N m_g}{1 + k/4\Xi\pi}\right)^{1/2} \qquad (62)$$

$$\nabla\psi_{corr} = -4\left(\frac{G_N m_g}{1 + 4\pi\Xi/k}\right). \qquad (63)$$

Obviously the addition of the last three equations equals $\Delta\varphi$, given by Eq. (33), hence Eq. (60) is identical to the lens equations given by Eq. (34).

5. Discussion

Since the appearance of Bekenstein's relativistic MOND gravitation, the long-time incompleteness of Milgrom's modified Newtonian dynamics seems

to be filled up. Now we can investigate the GL phenomenon, such as the deflection angle or time delay, in the MOND paradigm, which could not be done before.

In this paper,we have investigated the GL phenomenon under the approximations and presuppositions of

(*i*) The GL lens is assumed to be static, spherically symmetric and following the thin lens formalism,

(*ii*) The motions of light rays are described in the framework of a Schwarzschild lens (i.e. a point mass model),

(*iii*) The revised physical metric is obtained by adding a positive scalar field into the potential of the standard Schwarzschild metric in symmetric coordinates,

Under these presuppositions, we find that when θ is larger than θ_0 ($\equiv r_0/D_L$; the MOND length scale), the reduced deflection angle α will approach to a constant for a given mass. This special prediction is a feature of GL in T*e*V*e* S, which is similar to that obtained by Mortlock & Turner (2001) with an intuitional approach. This is no surprise because just like in GR, the deflection of photons is simply twice the deflection of a massive particle with the speed of light in T*e*V*e* S, and thus Mortlock & Turner (2001) started from the correct premise. However, there is still something that was unknown before the appearance of T*e*V*e* S. For a static spherically symmetric spacetime, the case $\mu \ll 1$, which yields a relation of Eq. (29), is valid only when $|\nabla\Phi| \ll (4\pi/k)^2\mathfrak{a}_0$. This condition is valid in the MOND regime when $|\nabla\Phi|$ goes up to a couple of orders above $\mathfrak{a}_0$, or equivalently ϱ_0 is around one order of magnitude below r_0. (Bekenstein 2004)

We should address that the criteria of mass discrepancy for GL effects in T*e*V*e* S consists with that of stellar dynamics. In other words, only when $\varrho_0 \gg kr_0/4\pi$, the "missing mass" shall appear. This corresponds to the demarcation of the high surface brightness (HSB) and low surface brightness LSB galaxies from the dynamical analysis (Sanders & McGaugh 2002).

When we apply the deflection angle law to magnification, we find that in T*e*V*e* S the difference in the magnifications of the two images in the point mass model depends on the lens mass and source positions, and is always larger than one. This differs from traditional gravitational lensing, which says that the difference must always be one. Tens of thousands of the multiple images lensings found by the Sloan Digital Sky Survey (SDSS)(Stoughton et al. 2002) might be applied to check this prediction. For microlensing, light curves in T*e*V*e* S at deep MOND regime also differ from that in GR. To observe the discrepancy, the sources have to be located about $z_s \geq 1$ (Mortlock & Turner 2001).

Concerning time delay, the result is even more exotic. Add an arbitrary positive scalar field into the primary Schwarzschild metric Bekenstein (2004) the resultant contributions of the scalar field will reduce rather than enhance the potential time delay. Unfortunately, this phenomenon is unmeasurable in GL systems. What we can determine is only the time delay between two images produced by the same source. Therefore, the opposite feature of the time delay in T*e*V*e* S, which is the same for all images, will be canceled out, and only those parts contributed from the deflection potential can be observed.

Even though the opposite effect on the potential time delay due to the scalar field can not be measured in GL system, the time delay between two images offers another constraint on the needed mass in GR and T*e*V*e* S, which usually differs from that given by the deflection angle. Actually the mass ratio obtained from these two approaches will be the same, otherwise the mass ratio given by time delay would always be smaller than that by deflection angle. In one word, the MOND and CDM paradigm are not mutually alternatives in a GL system, when we consider deflection angle and time delay at the same time.

The first time delay for the gravitational lens Q0957+561 was measured in 1984 (Florentin-Nielsen 1984), since then more than 11 time delay lenses have been found, including 7 systems with a good quality of the astrometric data and 2 systems with serious problems (Kochanek & Schechter 2003). It would be very interesting if those debatable systems can be explained in the framework of T*e*V*e* S. However, we emphasize that the measurement of time delay is controversial. Actually it took almost 20 years of debate between a short delay and a long delay on the first observed time delay source (Petters, Levine

& Wambsganss 2001).

It also worth noting that our conclusion is based on the Bekenstein's approach to investigate a gravitational lens. He assumes a *priori* that the potential dominating in the massive particle dynamics agrees with the potential influences on the GL system. In other words, the temporal and spatial components of the metric respectively have a form $-(1+2\Phi)$ and $(1-2\Phi)$, whence $\tilde{g}_{tt}\tilde{g}_{ii} \simeq 1$. However, there is still a chance that these two potentials are not the same and even that $\tilde{g}_{tt}\tilde{g}_{ii} \neq 1$ under the framework of T*e*V*e*S. If so, the conclusion of this paper may be different (Edery 1999; Bekenstein et al. 2000).

Since we can observe the influence of gravity on gravitational lens phenomena at distances further away than we can for the dynamics of massive particles, GL systems offer a good method to distinguish between GR and T*e*V*e*S. Although we need more than the point mass model to fit astrometric data, this simplified model has in principal told us the exotic predictions of the deflection angle and time delay. It would be very interesting if we could develop a more realistic model to fit the known data in GL systems.

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